premum; x, spatial variable; t, time; ∞, infinity; c, positive constant; π, number approximately equal to 3.14; $\partial/\partial t$, $\partial/\partial x$, partial derivatives with respect to time and space, respectively.

DETERMINATION OF THE CONTACT THERMAL RESISTANCE FROM THE SOLUTION OF THE INVERSE PROBLEM OF THERMAL CONDUCTIVITY

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The contact resistance at the boundary between an orthropic reinforcing rod and an isotropic matrix is determined from the solution of the inverse problem of thermal conductivity, using the gradient method. The suggested modification of the computational algorithm as the initial calculation of the initial period of the thermal process is shown to enhance the resolving power of the method and the choice of zeroth approximations from below is shown to ensure monotonic convergence of the solution.

One parameter which determines the heat exchange in reinforced materials or in elements of complex structures is the contact thermal resistance (CTR) due to the nonideal mechanical coupling of the contact surfaces. In theoretical studies on CTR the contribution of the thermal resistance to the heat transfer across the contact interface of the media is described by a condition in the form

$$\lambda_1 \frac{\partial T_1}{\partial n} = \lambda_2 \frac{\partial T_2}{\partial n}, \quad -\lambda_1 \frac{\partial T_1}{\partial n} R = T_2 - T_1,$$

where R is the contact thermal resistance, λ_1 and λ_2 are the thermal conductivities of the media in contact, and n is the normal to the contact surface. Thermal contact resistance has been considered as a function of the determining parameters, e.g., temperature [1], thermal stresses [2], and a complex of parameters in the form of the compression pressure, the instantaneous tensile strength, and the height of the irregularities [3]. Nevertheless, even though different determining parameters are chosen, the value of the thermal resistance for each specific case is determined experimentally or is approximated [4].

Artyukhin and Nenarokomov [5] advanced a fairly effective treatment for determining CTR as a function of the temperature on the basis of the solution of the inverse one-dimensional problem. In view of this, it is of some interest to extrapolate this treatment to the two-dimensional case and to study the possibilities of an algorithm for the solution.

The CTR is reconstructed on the example of an orthotropic cylindrical region surrounded by an isotropic medium. The mathematical simulation of the heat transfer in the media in contact was presented in the form of a two-dimensional, nonstationary system of equations involving the temperature dependence of the coefficient being sought:

$$\frac{\partial T_1}{\partial t} = a_1 \left(\frac{\partial^2 T_1}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T_1}{\partial r} \right) \right); \tag{1}$$

$$\frac{\partial T_2}{\partial t} = a_2 \frac{\partial^2 T_2}{\partial z^2} + a_3 \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T_2}{\partial r} \right);$$

$$T_1(z, r, 0) = T_2(z, r, 0) = T_0 = \text{const}; \tag{2}$$

$$\frac{\partial T_1(0, r, t)}{\partial z} = \frac{\partial T_2(0, r, t)}{\partial z} = 0;$$

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$$\frac{\partial T_1(L, r, t)}{\partial z} = \frac{\partial T_2(L, r, t)}{\partial z} = 0;$$

$$\frac{\partial T_2(z, 0, t)}{\partial r} = 0; \ \lambda_3 \frac{\partial T_2(z, R_1, t)}{\partial r} = \lambda_1 \frac{\partial T_1(z, R_1, t)}{\partial r};$$

$$-\lambda_3 \frac{\partial T_2(z, R_1, t)}{\partial r} R(T_2) = T_2(z, R_1, t) - T_1(z, R_1, t);$$

$$-\lambda_1 \frac{\partial T_1(z, R_2, t)}{\partial r} = q_0(z).$$

The parametrization of the CTR was reduced to a polynomial of

$$R(T_2) = \sum_{i=0}^{2} c_i T_2^i. \tag{3}$$

The inverse problem consisted in determining Eq. (3) from the condition for the minimum of the objective functional

$$I = \sum_{j=1}^{2} \int_0^{\tau_m} [T_j(z_j, r_j, t, R(T_2)) - f_j(t)]^2 dt,$$

where T and f are the model and experimental values of the temperature at the fixed points j = 1, 2 of the media in contact. The temperature measurements are assumed to be made along the axis of the region studied and in the central part of the isotropic material. The functional is minimized by the method of quickest descent [6]

$$c_i^{k+1} = c_i^k - \beta_k \frac{\partial I}{\partial c_i}, \tag{4}$$

where k is the number of the iteration, c_i^0 are the parameters of the zeroth approximation, and β_k is the depth of the descent. As the regularization parameter we take the number of iterations corresponding to the termination of the iteration cycle by the requirement that the level of discrepancy of the functional match the error of the experimental data. The procedure of moving to the next iteration according to (4) is carried out on the basis of the solution of auxiliary boundary-value problems obtained for the temperature increments in order to determine the depths of descent and for the sensitivity functions in order to determine the components of the gradient of the functions. The form of these problems is similar to the initial system of equations (1)-(2), but with singularities in the boundary conditions at the contact between the media:

– for the first problem

$$\lambda_3 \frac{\partial \Theta_2(z, R_1, t)}{\partial r} = \lambda_1 \frac{\partial \Theta_1(z, R_1, t)}{\partial r},$$

$$-\lambda_3 \frac{\partial T_2}{\partial r}\left(\frac{\partial R}{\partial T_2}\Theta_2 + \Delta R\right) - \lambda_3 \frac{\partial \Theta_2}{\partial r} R(T_2) = \Theta_2 - \Theta_1;$$

$$\Delta R = \sum_{i=0}^{2} \Delta c_i T_2^i; \ \Delta c_i = -\beta_k \frac{\partial I}{\partial c_i},$$

where Θ_1 and Θ_2 are the increments of temperature in the isotropic and orthotropic regions, respectively, and ΔR is the increment of the CTR;

– for the second problem

$$\lambda_3 \frac{\partial \Psi_{2i}(z, R_1, t)}{\partial r} = \lambda_1 \frac{\partial \Psi_{1i}(z, R_1, t)}{\partial r},$$

$$-\frac{\partial}{\partial c_i}\left(\lambda_3 \frac{\partial T_2}{\partial r} R(T_2)\right) = \Psi_{2i} - \Psi_{1i},$$

where Ψ_{1i} and Ψ_{2i} are sensitivity functions.

The inverse problem of thermal conductivity (IPTC) is solved numerically on the basis of the methods of subcomponent splitting and difference factorization [7]. The effectiveness of the algorithm was checked, first by solving the direct problem with a known dependence of the CTR on the temperature, after which the solution was used as the initial information for solving the inverse problem. As was to be expected, the iteration cycle was left on the basis of coincidence of neighboring iteractions in the case when the values of c_i^0 corresponding to the solution of the direct problem were taken as the zeroth approximation. We point out that the calculations are carried out for dimensionless temperature, which is obtained as the ratio of T to T_0. This makes it possible to dispense with order-of-magnitude checking of the values of c_i^0 in (3) and to reduce their choice to values of one order of magnitude.

In solving the IPTC, when the thermophysical characteristics are reconstructed as functions of the temperature, use is quite often made of the values of these coefficients at some initial temperature. When such information is lacking, however, as noted in [8], the determination of the functional relation itself depends strongly on the accuracy of this approximation, and in the given case this is c_i^0. Accordingly, Kolesnikov and Protod'yakonova [8] proposed an algorithm for two-stage determination of the coefficients: In the first stage the IPTC is solved on the assumption that the reconstructed characteristics are constant in time over the entire computing interval while in the second stage the same problem is solved, but with allowance for the temperature dependence of the coefficients and using the solution of the first stage as the zeroth approximation.

This algorithm was used in model calculations to determine the CTR for $\lambda_1 = 100$ W/m·K, $\lambda_2 = 150$, $\lambda_3 = 80$, $a_1 = 9.6\cdot10^{-5}$ m²/sec, $a_2 = 2.10^{-4}$, $a_3 = 10^{-4}$, $L = 2\cdot10^{-2}$ m, $R_1 = 10^{-3}$, and $R_2 = 3\cdot10^{-3}$; the thermal flux density $q_0(z)$ in this case varied exponentially from $3\cdot10^5$ to $3\cdot10^6$ W/m². A check of the effectiveness of the algorithm when the CTR as a function of the temperature varied within 10% revealed a satisfactory picture. The maximum deviation of the calculated temperature dependence of the CTR from the model dependence did not exceed 7%. Some difficulties with the algorithm are encountered when the CTR is markedly dependent on the temperature, e.g.,

$$R(T_2) = 10^{-3}\sum_{i=0}^{2} T_2^i, \quad . \tag{5}$$

The temperature factor in (5) reached a value of five because of the existence of longitudinal nonisothermicity and the coefficient R varied by almost an order of magnitude. When calculations were carried out in accordance with [8], it turned out that in the first stage the algorithm does not allow even an indirect solution to be obtained since there was no quitting of the iteration cycle, even though the discrepancy criterion was increased from $5\cdot10^{-3}$ to 1.0, which corresponded to allowing a 30% deviation of the calculated temperature from the model temperature.

At the same time the modification of the computational algorithm in the form of a computing interval in the first stage to the initial heating of the region studied gives a positive result both for a moderate dependence of the CTR on the temperature and for (5). When the first initial steps with respect to time and the value of the termination criterion ($5\cdot10^{-3}$) are incorporated into the calculation (this criterion corresponded to an allowed deviation of the calculated temperature from the model temperature by no more than 3%), we obtained the value $R = c_0 = 3.362\cdot10^{-3}$ in the first stage. The result of the second stage, with allowance for the temperature dependence of R and the use of the solution of the first stage as the zeroth approximation, reconstructed a polynomial in the form

$$R(T_2) = 3{,}228\cdot10^{-3} + 5{,}949\cdot10^{-5}T_2 + 8{,}526\cdot10^{-4}T_2^2,$$

where it was assumed that $c_1^0 = c_2^0 = 10^{-6}$.

Comparative analysis of the dependence obtained with the model dependence (5) showed that the maximum errors, not exceeding 20%, are observed in the initial and final portions of the computational period and that R is determined with complete reliability in the middle part. The effectiveness of the computational algorithm was also checked on the solution of the problem with perturbed data. The absolute deviations from the model thermal curve varied within 2-5% and were random. The results of our numerical analysis showed that as the amplitude of the perturbations in the initial data decreased, the relative error of the reconstructed CTRs decrease from 11 to 7% and this indicates the real convergence and stability of the solution. The proof of the uniqueness remains an open question.

The proposed modification of the computational algorithm nevertheless enhances the resolving power of the method and can be used to solve inverse problems of thermal conductivity with coefficients.

NOTATION

T_0, initial temperature; T_1 and T_2, temperatures of the isotropic and orthotropic regions, respectively; t, time; z, r, cylindrical coordinate system; a_1, a_2, and a_3, thermal diffusivities of the isotropic and orthotropic materials; q_0, thermal flux density; $R(T_2)$, contact thermal resistance; R_1 and R_2, inner and outer radii of the isotropic region; L, length of the complex cylindrical region; λ_1, thermal conductivity of the isotropic material; and λ_2 and λ_3, principal thermal conductivities of the isotropic material.

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EFFICIENT ORGANIZATION OF THE QUENCHING OF ROLLED PRODUCTS ON THE BASIS OF SOLUTION OF AN EXTERNAL INVERSE HEAT-CONDUCTION PROBLEM

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Results are presented from mathematical modeling of the thermal interaction of sprayed liquid with a surface. The results made it possible to determine the depth of hardening of a flat-rolled product.

One of the most promising methods of heat treatment for strengthening rolled products is intensive cooling of a hot surface with a disperse liquid produced by flat-jet nozzles [1]. Of particular importance in this operation is the cooling rate $\Delta T/\Delta\tau$ in the temperature range from the critical point corresponding to the beginning of austenite decomposition to the temperature at which its stability is minimal, i.e., within the range 850-450°C. It is evident that mathematical modeling of the quenching process is impossible without reliable information on heat-transfer conditions and thermophysical characteristics (TPC) of the metal. A method was described in [2] to determine boundary conditions in the cooling of a surface by a sprayed liquid on the basis of solution of an external inverse heat-conduction problem (ICP) with the use of an iterative filter. The same study demonstrated the possibility of simultaneous identification of the TPC and boundary conditions by the solution of a combination ICP.

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